Vector functions

LHW 2 KEY

unless noted, some work required.

6 (3 pts)

Direction vector is

$\langle 2,3,1\rangle - \langle 1,0,1\rangle = \langle 1,3,0\rangle$

The line is (vector)

$$\boxed{\vec{r}(t) = \langle 2,3,1\rangle - \langle 1,3,0\rangle\cdot t}$$

(parametric)

$$\begin{cases} x = 2-t \\ y = 3-3t \\ z = 1 \end{cases} \qquad \text{where } t\in[\underset{t_1}{0},\underset{t_2}{1}]$$

Other param-s are possible: check that $\vec{r}(t_1) = \langle 1,0,1\rangle$ and $\vec{r}(t_2) = \langle 2,3,1\rangle$, or vice versa.

9 (4 pts)

Collision detection:

$$\begin{cases} t = 1+2t \\ t^2 = 1+6t \\ t^3 = 1+14t \end{cases} \quad \Rightarrow t=-1 \quad \Rightarrow (-1)^2 = 1-6 \Rightarrow \text{no solution} \Rightarrow \boxed{\text{no collision.}}$$

Intersection detection:

$$\begin{cases} t_1 = 1+2t_2 \\ t_1^2 = 1+6t_2 \\ t_1^3 = 1+14t_2 \end{cases} \Rightarrow (1+2t_2)^2 = 1+6t_2 \Rightarrow$$

$\Rightarrow 1+4t_2+4t_2^2 = 1+6t_2$

$\Rightarrow 4t_2^2 - 2t_2 = 0 \Rightarrow t_2 = 0$

OR $t_2 = \frac{1}{2}$. Then

we have 2 points of intersection: $(\underset{t_1}{1}, \underset{t_2}{0})$, and $(2, \frac{1}{2})$, which correspond to $\boxed{\langle 1,1,1\rangle}$ and $\boxed{\langle 2,4,8\rangle}$

Derivs and Integrals

6 (2 pts)

$$\vec{r}(t): \begin{cases} x = \ln t \\ y = 2\sqrt{t} \\ z = t^2 \end{cases}$$

then $\bar{r}'(t) = \left\langle \frac{1}{t}, \frac{1}{\sqrt{t}}, 2t \right\rangle$,

$$\bar{r}'(1) = \langle 1, 1, 2 \rangle$$

tangent line: $\boxed{\begin{cases} x = 0 + t \\ y = 2 + t \\ z = 1 + 2t \end{cases}}$

vector eq. OK too.

7 (4 pts)

$$\begin{cases} t = 3 - s \\ 1 - t = s - 2 & \Rightarrow 1 - (3 - s) = s - 2 \Rightarrow ? \\ 3 + t^2 = s^2 & \Rightarrow 3 + (3 - s)^2 = s^2 \Rightarrow \end{cases}$$

$\Rightarrow 3 + 9 - 6s + s^2 = s^2 \Rightarrow 6s = 12 \Rightarrow s = 2$

$\Rightarrow t = 1$, intersection at

$$\boxed{\langle 1, 0, 4 \rangle}$$

angle of intersection is the angle between $\bar{r}_1'(1)$ and $\bar{r}_2'(2)$.

$\bar{r}_1(1) = \langle 1, -1, 2 \rangle$, $\bar{r}_2'(2) = \langle -1, 1, 4 \rangle$,

$$\cos\theta = \frac{\bar{r}_1'(1) \cdot \bar{r}_2'(2)}{|r_1'| \cdot |r_2'|} = \frac{-1 - 1 + 8}{\sqrt{6} \cdot \sqrt{18}} = \frac{1}{\sqrt{3}}$$

$$\boxed{\theta \approx 54.74^\circ}$$

9

2 pts

$$\int \bar{r}'(t)\,dt = \langle \int t\,dt, \int e^t dt, \int te^t dt \rangle$$

$$= \langle \frac{t^2}{2}, e^t, te^t - e^t \rangle + \langle c_1, c_2, c_3 \rangle = \vec{r}(t)$$

But $\vec{r}(0) = \langle 0, 1, -1 \rangle + \langle c_1, c_2, c_3 \rangle = \langle 1, 1, 1 \rangle$ so

$c_1 = 1$, $c_2 = 0$, $c_3 = 2$, and

$$\boxed{\vec{r}(t) = \langle \frac{t^2}{2} + 1, e^t, te^t - e^t + 2 \rangle}$$

Motion in space

3

3pts

$\bar{v}(t) = \bar{r}'(t) = \boxed{\langle e^t, 2e^{2t} \rangle}$ - velocity

$\bar{a}(t) = \bar{v}'(t) = \boxed{\langle e^t, 4e^{2t} \rangle}$ - accel.

$Sp(t) = |\bar{v}(t)| = \sqrt{(e^t)^2 + (2e^{2t})^2} =$

$= \sqrt{e^{2t} + 4e^{4t}} = e^t\sqrt{1 + 4e^{2t}}$ - speed

ignore drawing.

4

3 pts.

$\bar{r}(t) = \langle t, 2\cos t, \sin t \rangle$

$\bar{v}(t) = \bar{r}'(t) = \langle 1, -2\sin t, \cos t \rangle$ - velocity

$\bar{a}(t) = \bar{v}'(t) = \langle 0, -2\cos t, -\sin t \rangle$ - accel.

$s_p(t) = |\bar{v}(t)| = \sqrt{1 + 4\sin^2 t + \cos^2 t} =$

$= \sqrt{2 + 3\sin^2 t}$ - speed

ignore drawing.