Derivatives

$\boxed{3}$ $\boxed{3\text{pt}}$ ⓐ $\nabla f = \langle f_x, f_y, f_z \rangle = \left\langle \frac{1}{2\sqrt{x+yz}}, \frac{z}{2\sqrt{x+yz}}, \frac{y}{2\sqrt{x+yz}} \right\rangle$

ⓑ $\nabla f(1,3,1) = \left\langle \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle$

ⓒ $|\vec{u}| = \left|\left\langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \right\rangle\right| = 1$, so

$D_{\bar{u}} f(1,3,1) = \nabla f \circ \bar{u} = \left\langle \frac{1}{4}, \frac{1}{4}, \frac{3}{4} \right\rangle \circ \left\langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \right\rangle =$

$= \frac{23}{28}$

answer OK

$\boxed{4}$ $\boxed{3\text{pt}}$ Let $\vec{u} = \frac{\langle 1,2,3 \rangle}{|\langle 1,2,3 \rangle|} = \frac{\langle 1,2,3 \rangle}{\sqrt{14}}$

$D_{\bar{u}} f(4,1,1) = \nabla f \circ \bar{u} = \left\langle \frac{1}{2}, -1, -1 \right\rangle \circ \frac{1}{\sqrt{14}} \langle 1,2,3 \rangle$

$= \frac{1}{\sqrt{14}} \left(\frac{1}{2} - 2 - 3 \right) = \frac{9}{2\sqrt{14}}$

answer OK

⑥ 2 pt

$$\nabla f = \left\langle \frac{1}{\cos^2(x+2y+3z)}, \frac{2}{\cos^2(x+2y+3z)}, \frac{3}{\cos^2(x+2y+3z)} \right\rangle$$

$\nabla f(-5,1,1) = \langle 1, 2, 3 \rangle$ - direction (any multiple)

$|\nabla f| = \sqrt{1+4+9} = \sqrt{14}$ - rate of change.

⑦ 2 pt

$\nabla f = \langle 2x + y\cos(xy), x\cos(xy) \rangle$

$\nabla f(1,0) = \langle 2, 1 \rangle$

$$\begin{cases} \langle 2,1 \rangle \circ \langle u_1, u_2 \rangle = 1 \\ u_1^2 + u_2^2 = 1 \end{cases}$$

$2u_1 = 1 - u_2 \iff u_2 = 1 - 2u_1$

$u_1^2 + (1-2u_1)^2 = 1$

$u_1^2 + 1 - 4u_1 + 4u_1^2 = 1$

$5u_1^2 - 4u_1 = 0$

$u_1(5u_1 - 4) = 0 \Rightarrow u_1 = 0$ or $\frac{4}{5}$

$u_2 = 1$ OR $-\frac{3}{5}$

$\langle 0, 1 \rangle$ and $\langle \frac{4}{5}, -\frac{3}{5} \rangle$.

(11) let $F(x,y,z) = 1 - x^2 - 2y^2 - 3z^2 = 0$

[2pt]

$\nabla F = \langle -2x, -4y, -6z \rangle$

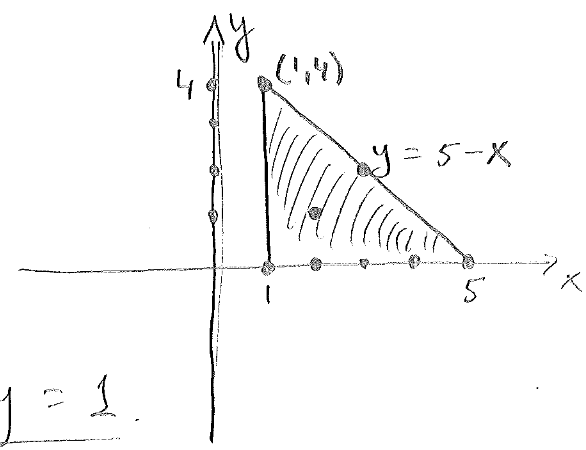
$$\begin{cases} -2x = 3c \\ -4y = -c \\ -6z = 3c \\ x^2 + 2y^2 + 3z^2 = 1 \end{cases} \qquad \begin{cases} x = \frac{-3c}{2} \\ y = \frac{c}{4} \\ z = -\frac{c}{2} \\ \text{---} '' \text{---} \end{cases}$$

$$\frac{9c^2}{4} + 2\frac{c^2}{16} + 3\frac{c^2}{4} = 1$$

$$c^2 = \frac{8}{25} \Rightarrow c = \pm \frac{2\sqrt{2}}{5}$$

$$\pm \left\langle \frac{-3\sqrt{2}}{5}, \frac{\sqrt{2}}{10}, -\frac{\sqrt{2}}{5} \right\rangle$$

Max & Min



(6) crit. pts. of f:

[4pt]

$$\left.\begin{aligned} f_x &= y - 1 = 0 \\ f_y &= x - 2 = 0 \end{aligned}\right\} \Rightarrow \underline{x = 2,\ y = 1.}$$

For $x = 1$, $h(y) = f(1,y) = 2 - y$, linear $\Rightarrow$

no crit. pts.

For $y = 0$, $h(x) = f(x,0) = 3 - x$, linear.

For $y = 5 - x$, $h(x) = f(x, 5-x) =$

$= 3 + \underline{x(5-x)} - \underline{x} - 2(5 - \underline{x}) = 6x - 7 - x^2$

vertex at $x = 3, \quad y = 2$.

$f(2,1) = 1$

$f(3,2) = 2$

$f(1,0) = 2$

} global max.

$f(1,4) = -2$

$f(5,0) = -2$

} global min

8 2pt

let (x,y,z) be on the surface.

then distance to origin

$$d(x,z) = \sqrt{x^2+y^2+z^2} = \sqrt{x^2+9+xz+z^2}$$

minimizing d by minimizing $D(x,z) = [d(x,z)]^2$

$$D(x,z) = x^2+9+xz+z^2$$

$$\left.\begin{aligned} D_x &= 2x+z = 0 \\ D_z &= x+2z = 0 \end{aligned}\right\} \Rightarrow x, z = 0.$$

$(0, \pm 3, 0)$